

OCR

A Level

Computer
Science

H446 – Paper 1



Binary arithmetic

Unit 6
Data types



PG ONLINE

Objectives

- Use sign and magnitude to represent negative numbers in binary
- Use two's complement to represent negative numbers in binary
- Add and subtract binary integers
- Represent fractions in fixed point binary

Arithmetic and comparison operators

- All computer instructions are based on the fundamental arithmetic and comparison operators
 - What are these operators?

Arithmetic and comparison operators

- Arithmetic operators:
 - Addition, subtraction, multiplication, division, DIV, MOD, exponentiation
- Comparison operators:
 - Equal, not equal, greater than, less than, greater than or equal, less than or equal (=, <>, >, <, >=, <=)

Binary addition

$$1 + 1 = 10?$$

- Computers are built from combinations of electronics called logic gates
- The most fundamental operation a computer processor performs is adding values
- Binary values are input into the processor and

Simple Addition

- The result of an addition can have a value and a **carry** when the result is too large for the number position it is in
- Working right to left :

1. Add the Units

2. If Over 9, *Carry* Tens

3. Add Tens

$$\begin{array}{r} 1 \\ 19 \\ + 17 \\ = 36 \end{array}$$

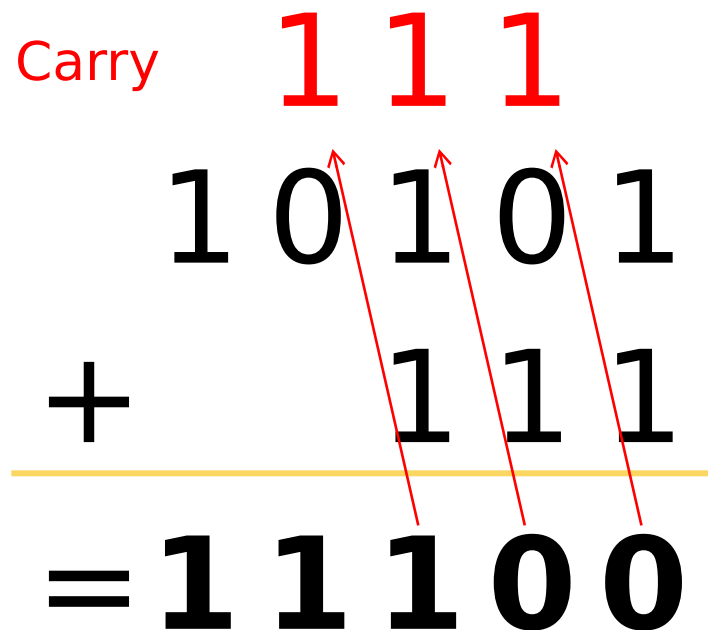
Binary addition

- The rules of binary addition are the same:
 - Three 1s occur during a carry operation

0	0	1	1	1
+ 0	+ 1	+ 0	+ 1	+ 1
0	1	1	1 0	1 1

Adding binary values

- This concept can be extended to binary values of more than one bit

$$\begin{array}{r} \text{Carry} \quad 1 \quad 1 \quad 1 \\ 1 \quad 0 \quad 1 \quad 0 \quad 1 \\ + \quad \quad 1 \quad 1 \quad 1 \\ \hline = 1 \quad 1 \quad 1 \quad 0 \quad 0 \end{array}$$


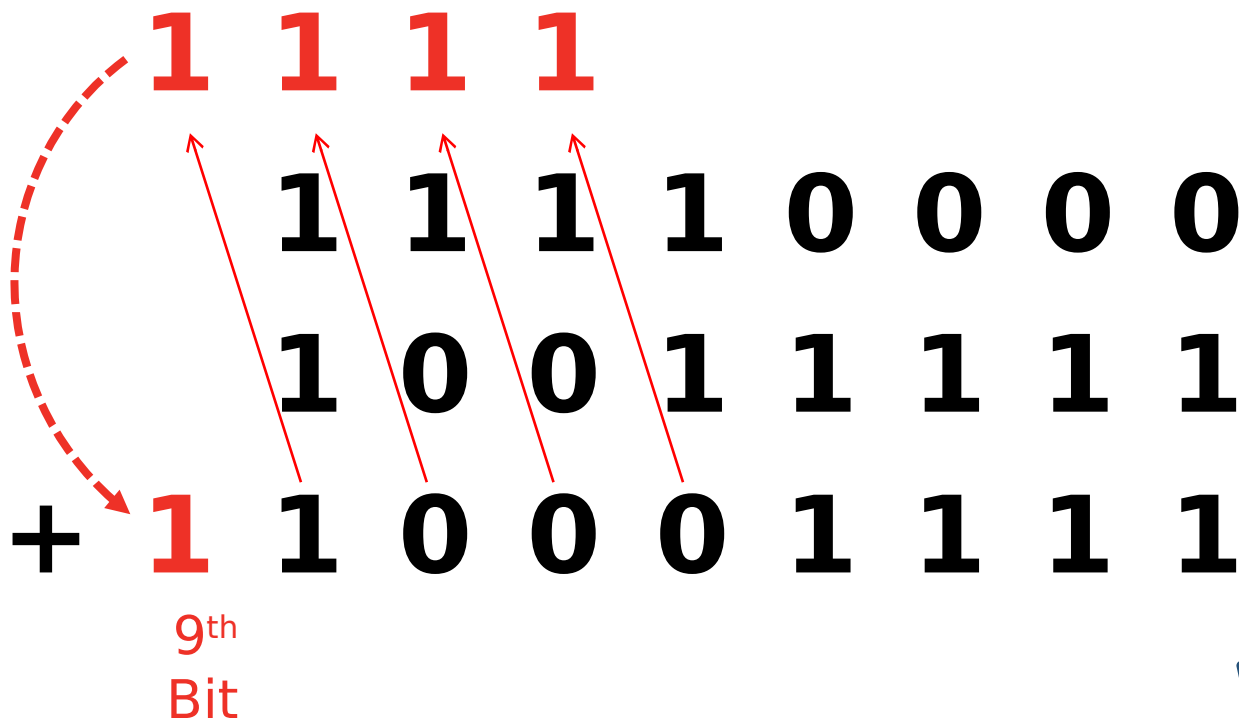
Adding bytes

- Computers work with a fixed number of bits at a time
 - This can cause problems
 - What problem will there be when adding the following bytes and storing the result in one byte?

$$\begin{array}{r} 11110000 \\ + 10011111 \\ \hline \end{array}$$

Overflow error

- When the result of an addition is too large for the number of bits the computer works with, there will be an **overflow error**



Binary addition

- Complete **Worksheet 3, Task 1**



Calculating range

- Remember that the number of bits determines how many values can be represented in binary
 - How many binary values can be represented in 4 bits?
 - What is the highest and lowest values that can be represented?

0 . . . $2^n - 1$

Representing negative numbers

- How can negative numbers be represented?
- One method is called **sign and magnitude**
- The first bit acts as the sign, 0 for + and 1 for -
- How would the number -3 be represented in an 8-bit byte using sign and magnitude?

Sign and magnitude

- The binary representation of 3 is 00000011
- The binary representation of -3 is 10000011
- Add these two numbers:

$$\begin{array}{r} 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1 \\ +\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1 \\ \hline =\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 0 \end{array}$$

- This is -6!
- Clearly arithmetic does not work in the normal way using sign and magnitude

Two's complement

- Negative values are most commonly represented using a method called **two's complement**
 - Similar to an analogue counter, turning the wheel back 1 will equal 0000



- Turning back 1 from 00000000 is 11111111
- 11111111 is -1 in binary

Two's complement

- The two's complement of a signed binary value is found by flipping all of the bits and adding one

11111101 = -3

11111110 = -2

11111111 = -1

00000000 = 0

00000001 = 1

00000010 = 2

00000011 = 3

Signed binary representation

- Find -77_{10} in two's complement:

- Find the positive value first:

128	64	32	16	8	4	2	1	
0	1	0	0	1	1	0	1	=77

- Flip the bits:

1	0	1	1	0	0	1	0
---	---	---	---	---	---	---	---

- And add 1:

1	0	1	1	0	0	1	1	=	⁻ 77
---	---	---	---	---	---	---	---	---	-----------------

Two's complement range

- Note that the number of values that can be represented is the same as for positive binary numbers
- However, half the range of values now represents negative numbers
 - Maximum for 8 bits is 127_{10} which is 01111111 in binary
 - Minimum is -128_{10} which is 10000000 in binary

$$-(2^{(n-1)}) \dots 2^{(n-1)} - 1$$

- What range of values can be represented in 16 bits?



Subtraction with two's complement

- Two's complement can be used to ensure subtraction will occur when adding negative values
- The calculation $65_{10} - 43_{10}$ should produce the same result as $65_{10} + -43_{10}$
- This way we can always use addition when we want to subtract

Binary subtraction

- Using two's complement we can find $65_{10} + -43_{10}$

$$\begin{array}{r} \\ \\ \\ + \\ \hline \text{Ignore the carry bit} \end{array}$$

Answer is 22_{10}

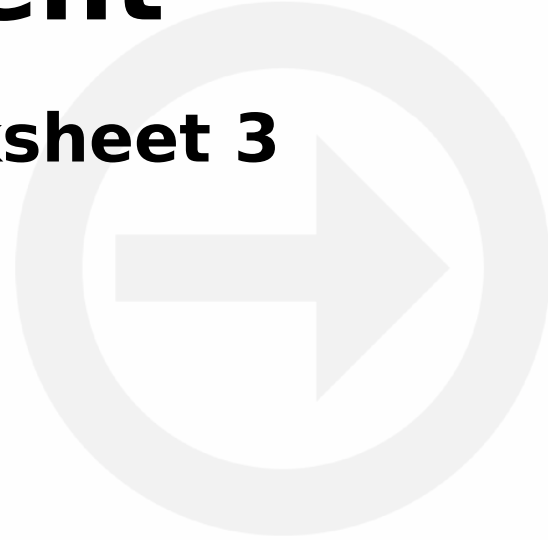
Two's complement overflow

- Where the result of the addition is too large to fit in the allocated number of bits, overflow will occur
 - This is the same as standard binary addition
 - If the result requires more bits than are available, the number will not be represented correctly
 - A '1' in the left most bit indicates a negative number, therefore overflow occurs above 127 with 8 bits

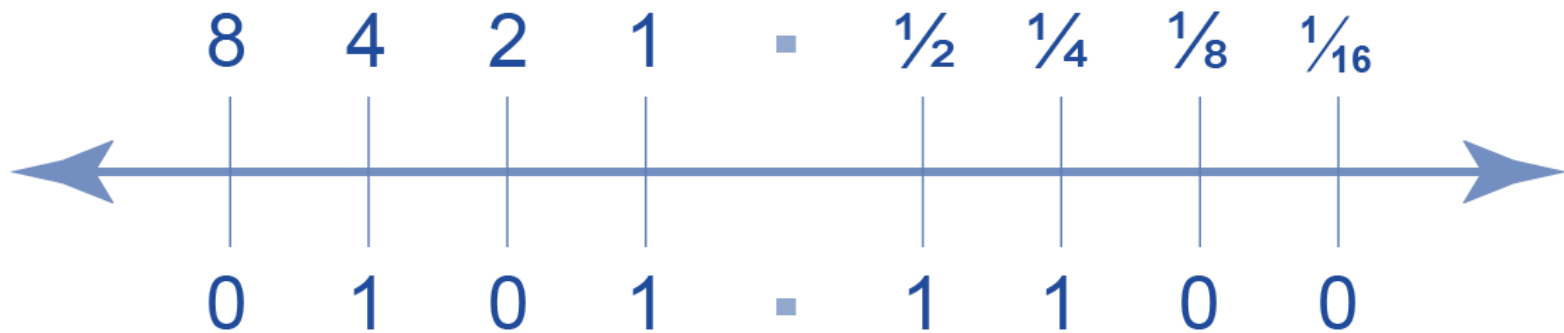
11010101

Two's complement

- Complete **Task 2** on **Worksheet 3**



- Using bits to the right of the units column (after a notional point) introduces fractional values



Fractional values

- Fractional values are negative powers of 2
- For example:
 - 2^{-2} is the same as $\frac{1}{2^2}$ or 0.25_{10}

Place value	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Decimal equivalent	4	2	1	0.5	0.25	$\frac{0.125}{5}$

Fixed-point binary

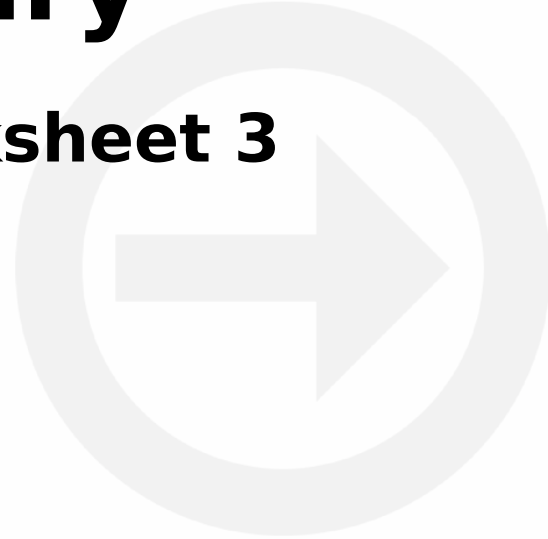
- A fixed-point binary value uses a specified number of bits where the placement of the binary point is fixed
 - For example, in an 8 bit fixed-point binary value, the binary point could be set between the fourth and fifth bits

2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}
8	4	2	1	●	0.5	0.25	0.125	0.0625
0	1	1	0		1	0	1	0

- What effect does this have on accuracy and range?

Fixed point binary

- Complete **Task 3** on **Worksheet 3**



Plenary

- The fundamental building block of processor operation is addition
- Negative values use two's complement in order to be processed correctly without the need for additional processor logic circuits
- Fixed point binary is one way of representing binary fractions

Copyright

© 2016 PG Online Limited

The contents of this unit are protected by copyright.

This unit and all the worksheets, PowerPoint presentations, teaching guides and other associated files distributed with it are supplied to you by PG Online Limited under licence and may be used and copied by you only in accordance with the terms of the licence. Except as expressly permitted by the licence, no part of the materials distributed with this unit may be used, reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic or otherwise, without the prior written permission of PG Online Limited.

Licence agreement

This is a legal agreement between you, the end user, and PG Online Limited. This unit and all the worksheets, PowerPoint presentations, teaching guides and other associated files distributed with it is licensed, not sold, to you by PG Online Limited for use under the terms of the licence.

The materials distributed with this unit may be freely copied and used by members of a single institution on a single site only. You are not permitted to share in any way any of the materials or part of the materials with any third party, including users on another site or individuals who are members of a separate institution. You acknowledge that the materials must remain with you, the licencing institution, and no part of the materials may be transferred to another institution. You also agree not to procure, authorise, encourage, facilitate or enable any third party to reproduce these materials in whole or in part without the prior permission of PG Online Limited.